

# Generation of Coalition Structures to Provide Proper Groups' Formation in Group Recommender Systems

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## ABSTRACT

Group recommender systems usually provide recommendations to a fixed and predetermined set of members. In some situations, however, there is a set of people ( $N$ ) that should be organized into smaller and cohesive groups, so it is possible to provide more effective recommendations to each of them. This is not a trivial task. In this paper we propose an innovative approach for grouping people within the recommendation problem context. The problem is modeled as a *coalitional game* from Game Theory. The goal is to group people into exhaustive and disjoint coalitions so as to maximize the *social welfare* function of the group. The optimal *coalition structure* is that with highest summation over all *social welfare* values. Similarities between recommendation system users are used to define the *social welfare* function. We compare our approach with K-Means clustering for a dataset from Movielens. Results have shown that the proposed approach performs better than K-Means for both *average group satisfaction* and *Davies-Bouldin index* metrics when the number of coalitions found is not greater than 4 ( $K \leq 4$ ) for a population size of 12 ( $N = 12$ ).

## Categories and Subject Descriptors

H.3.3 [Information Search and Retrieval]: Information Filtering; D.2.8 [Software Engineering]: Metrics—*complexity measures, performance measures*

## General Terms

Algorithms, Experimentation

## Keywords

Group Recommendation, Group Formation, Game Theory, Coalitional Games, Social Welfare

## 1. INTRODUCTION

Recommendation systems perform personalized recommendation of items that are of potential interest for such a user [1]. Based on the previously evaluated items, a recommendation system builds a user's profile. The similarity of such profile with other users' profiles (*collaborative filtering*) is thus used to recommend items to the user [4].

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In some contexts, where groups of people share location or interests, recommendation for individuals may be not that appropriate. Recommending a restaurant for a business lunch, recommendation of a travel destination for a group of friends or a movie recommendation for a family are examples of scenarios where recommendation for a group is more appropriate. In such case, preferences of all group members are considered in order to perform recommendation. Among the difficulties that arise is the fact that group members may have different preferences or even conflicting ones. In order to solve this problem, the approach that has received most attention in scientific literature is the *aggregation of individual preferences* [11, 9].

Most group recommender systems assumes a fixed and predetermined set of members. In some situations, however, it is not appropriate or even not possible to previously establish a group. In the context of recommender systems, a set of people may be organized into smaller and cohesive groups, so it is possible to provide more effective recommendations to each of them.

There are other reasons that support the idea behind the generation of casual groups: (i) people often change their minds, which may justify eventual regrouping and (ii) technological issues might constrain the number of groups as well as the group size.

In this paper we propose an approach for grouping people in order to enable proper recommendation. The problem is modeled as a *coalitional game* from Game Theory. The goal is to group people into disjoint coalitions in such a way the *social welfare* function of the group is maximized. The optimal *coalition structure* is thus that with highest summation over all *social welfare* values. The *social welfare* function is defined upon the already known similarities between users' profiles. This means that more homogeneous groups tend to receive better recommendations.

There are some related work concerning group formation in the scientific literature [3, 5]. In [13], for instance, personality and confidence are used to define coalitions and achieve agreements within the group. The authors claim that the approach seeks to reproduce the behavior of real users during negotiation of items for consumption, trying to identify group leaders and testing the number of people they are able to convince. The author further believes that user's personality determines whether its choices are more assertive and cooperative in regards to the group. Another approach [5] automatically discovers communities of interest (e.g. a group of individuals who share and exchange ideas of particular interest) and produces recommendations for them.

For that, it considers ontology-based profiles. Each profile measures the user's interest in the ontology concepts. Then, these interests are used to form clusters of ontology concepts. As a result, relationships between users on different levels are formed providing a multi-level interest network which allows for finding multiple communities of interest. [3] proposes an algorithm which identifies groups of users whose preferences are similar and recommends items for these groups. Groups of different granularities are created by an algorithm for automatic detection of communities based on modularity. Experimental results show that the quality of the recommendation for the group increases linearly with the number of groups created.

The rest of the paper is organized as follows. Section 2 reviews group recommender systems. Section 3 reviews the *Cooperative Game Theory*, since it is the theoretic basis of the proposed approach. In Section 4, we provide a formal definition to the problem of group formation as a coalitional game. Section 5 describes the experiments performed discuss the results. Finally, in Section 6 we present some concluding remarks and further work.

## 2. GROUP RECOMMENDER SYSTEM

Although recommendation systems traditionally recommend items for individual users, there is an increasing amount of research focused on recommendation for groups of users [12, 9]. In such case, recommendations aim to satisfy a group of users with potentially conflicting interests.

The need for choosing a method of aggregation to generate recommendations is the key characteristic of group recommendation. Although different aggregation strategies differ in the way they manipulate and represent users' preferences, virtually all of them adopts one of three schemes: (1) aggregates a single set of individual recommendations, (2) builds a unique representation model for the group, or (3) aggregate the ratings/preferences for particular items.

Regarding the problem of groups formation in the context group recommender systems, a group may be characterized as [3, 2]:

- **Established Group:** a number of people who have explicitly decided to be part of a group, due to some long-term shared interest;
- **Occasional Group:** a number of people doing something together occasionally. Members have a common purpose at a particular time;
- **Random Group:** a number of people who share the same environment at a particular time, with no clear motivation to be tied;
- **Automatically Identified Group:** groups that are automatically detected considering members' preferences and/or available resources.

A large set of different aggregation functions for individual preferences defined in the *Social Choice Theory* are known in the literature of group recommender systems. *Average* is one of the most well known aggregation strategies. It assumes equal importance to group members and computes the average of the group evaluation for the items. The disadvantage of this strategy is due to the heavy reliance on group size. For groups with fewer members, for instance,

each member opinion has a greater impact on the average. The *Average* function seeks to find the central tendency of a sample (equation 1).

$$\hat{p}(g, i) = \frac{1}{|g|} \sum_{u \in g} \hat{p}(u, i) \quad (1)$$

where  $\hat{p}(u, i)$  is the predicted evaluation for each user  $u$  and every item  $i$ .  $\hat{p}(g, i)$  is the final prediction value of item  $i$  to the group.

Each aggregation strategy has advantages and disadvantages, so some comparison criteria has been defined [9]. One of them consists in to maximize the *average group satisfaction*. Previously, it is necessary to calculate the average individual satisfaction for recommended items (equation 2).

$$S(u, R) = \frac{1}{|R|} \sum_{i \in R} \hat{p}(u, i) \quad (2)$$

where  $R$  is the set of recommended items and  $\hat{p}(u, i)$  is the predicted evaluation value of user  $u$  for item  $i \in R$ .

The value of the average group satisfaction to recommendation  $R$  is computed by Equation 3.

$$S(g, R) = \frac{1}{|g|} \sum_{u \in g} S(u, R) \quad (3)$$

where  $g$  is the user group and  $S(u, R)$  is the average satisfaction of each group member for recommendation  $R$ .

## 3. COOPERATIVE GAME THEORY

In *Game Theory* [15], a *cooperative game* [14] is a game where a group of players, a *coalition*  $C$ , may enforce cooperative behavior. The game is thus a competition between coalitions of players rather than a competition between individual players.

A cooperative game comprises a set of  $n$  players (typically  $n > 2$ ) and  $Ag = \{1, \dots, n\}$  as the set of all these players. The subsets of  $Ag$  are possible coalitions. The *great coalition* is a coalition where  $C = Ag$ .

Formally, a cooperative is a tuple

$$\mathcal{G} = \langle Ag, v \rangle,$$

where  $Ag$  is a set of players and

$$v : 2^{Ag} \rightarrow \mathbb{R}$$

is called the *characteristic function* of the game. The *characteristic function* of the game assigns to each possible coalition a numeric value that intuitively represents the utility (or *payoff*) that can be distributed among coalition members.

If the entire system is known as *single designer*, the performance of individual players may not be relevant. Instead, it is concerned with maximizing the system's *social welfare*, i.e., maximizing the summation over individual coalitions.

A *coalition structure* is a partition of the full set of players  $Ag$  on mutually disjoint coalitions. From the set  $Ag = \{1, 2, 3\}$ , for instance, seven different coalitions

$$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}$$

and five different *coalition structures*  $\{\{1\}, \{2\}, \{3\}, \{\{1\}, \{2, 3\}\}, \{\{2\}, \{1, 3\}\}, \{\{3\}, \{1, 2\}\}, \{\{1, 2, 3\}\}$ .

may be provided.

Given a *coalitional game*  $\mathcal{G} = \langle Ag, v \rangle$ , an *optimal coalition structure*  $CS^*$  associated to  $\mathcal{G}$  is given by:

$$CS^* = \arg \max_{CS \in \text{partitions de } Ag} V(CS) \quad (4)$$

where

$$V(CS) = \sum_{C \in CS} v(C) \quad (5)$$

consists of the summation over the utility for each coalition from a given coalition structure.

## 4. PROBLEM FORMALIZATION

Let  $I = \{i_1, i_2, \dots, i_n\}$  and  $U = \{u_1, u_2, \dots, u_k\}$  be the set of all items and all users, respectively. Consider a set  $G$  of all groups with at least two members that may be formed by  $U$  and so,  $|G| = 2^k - k - 1$ . Consider, finally,  $g \in G$  and  $|g|$  defined as the number  $m$  of group members  $g$ . If, for instance, a group consists of users  $u_1, u_2$  and  $u_3$ , thus this can be expressed as  $g = \{u_1, u_2, u_3\}$  and  $|g|$ .

Let us assume  $p(u, i)$  as the evaluation for the item  $i$  user-supplied  $u$ , and  $p(u, i) = 0$ , if user  $u$  did not evaluate the item  $i$ . Consider  $\hat{p}(u, i)$  as the predictive assessment of item  $i$  for user  $u$ . The predicted evaluation  $\hat{p}(u, i)$  is obtained from a prediction function  $\hat{p}$  which considers, in turn, the similarity between items  $s : I \times I \rightarrow \mathbb{R}$  or the similarity between users  $s : U \times U \rightarrow \mathbb{R}$ .

*Backtracking* technique is used to find all possible combinations of users to form groups. The algorithm performs in  $O(2^n)$ .

The social welfare function to compute the payoff for each group is shown in Equation 6:

$$f = n^\alpha + \left( \frac{\sum (sim(u_i, u_j))}{\frac{n^2 - n}{2}} \right), \quad (6)$$

where  $n$  is the number of group members and  $sim(u_i, u_j)$  is the cosine similarity between two members (Equation 7):

$$sim(u_i, u_j) = \cos(\theta) = \frac{u_i \cdot u_j}{\|u_i\| \|u_j\|} \quad (7)$$

The result of the similarity lies in the interval  $[-1, 1]$ , where  $-1$  means users have opposite profiles,  $1$  means same profiles, and  $0$  indicates independence or orthogonality. Intermediate values indicate some similarity or dissimilarity levels. Similarity is calculated between all group members so to provide a similarity matrix, which is symmetric and the values of main diagonal equals to 1.

The summation is performed over the  $n^2 - n$  values above the main diagonal. The arithmetic mean favors groups with greater similarity between group members.

The factor  $n^\alpha$  is used to enable the variation on how the size of the group weights its *payoff*. A higher  $\alpha$  encourages the formation of larger groups and so, *coalition structures* with fewer elements. Conversely, lower  $\alpha$  values encourages the formation of smaller groups and thus, *coalition structures* with more elements. For  $\alpha = 1$  group *payoff* is independent of the group size.

Recommendation procedure selects items belonging to the intersection of the lists with rating predictions for each group member, individually (Equation 8):

$$H = \bigcap_{u=1}^n \{i \mid \hat{p}(u, i) \neq \emptyset, i \in I\} \quad (8)$$

In such approach, the game should answer the question *Which coalitions can be formed?*. As a consequence, the system assumes the role of *single designer* and does not need to deal with other cooperative games issues like *How the payoff should be distributed among the coalition members?*

*Backtracking* is once again used to find the whole set of *coalition structures* in order to find that which maximizes the summation over the payoff of all its coalitions ( $CS^*$ ). This performs in  $O(2^k)$ , with  $k$  being the number of possible coalitions.

As an illustrative example, consider the table 1 which contains six individual preferences and their ratings for five different items.

**Table 1: Individual user preferences.**

	A	B	C	D	E
Peter	1	2	3	3	2
Jane	5	3	5	3	4
Mary	5	2	5	3	1
Paul	1	5	1	2	1
John	1	1	4	1	3
Kate	3	5	3	2	3

Based on this table, we calculate the similarity between all users. The similarity matrix below is thus produced, where each cell represents the similarity between two users.

$$\begin{pmatrix} 1.00 & 0.90 & 0.84 & 0.75 & 0.87 & 0.87 \\ 0.90 & 1.00 & 0.94 & 0.68 & 0.88 & 0.92 \\ 0.84 & 0.94 & 1.00 & 0.60 & 0.78 & 0.82 \\ 0.75 & 0.68 & 0.60 & 1.00 & 0.50 & 0.90 \\ 0.87 & 0.89 & 0.78 & 0.50 & 1.00 & 0.78 \\ 0.87 & 0.92 & 0.82 & 0.90 & 0.78 & 1.00 \end{pmatrix}$$

Considering  $\alpha = 1$ , the calculation of the social welfare function for all groups and all possible coalition structures produce the following optimal coalition structure as a result.

$$\{\{Peter, John\}, \{Jane, Mary\}, \{Paul, Kate\}\}$$

## 5. EXPERIMENTS

Experiments have been performed in order to compare the proposed approach with K-Means clustering algorithm.

### 5.1 Dataset

The MovieLens <sup>1</sup> datasets are widely used in group recommendation research even though they do not actually provide information on groups. This is due to the complete lack of other well established and more suitable bases with groups information. The chosen dataset consists of 943 users, 1,682 movies and 100,000 movie ratings in the  $[1, 5]$  interval.

<sup>1</sup><http://www.grouplens.org/node/73/>

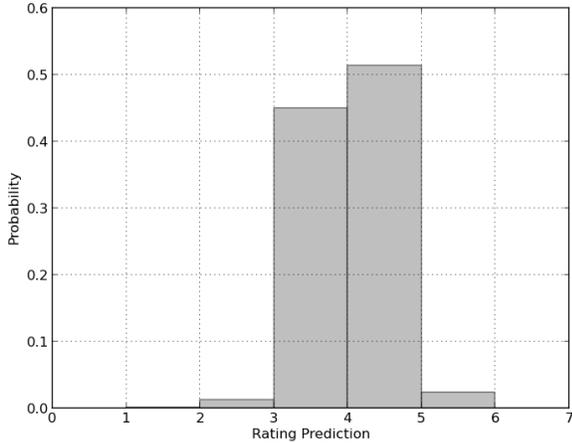


Figure 1: Histogram of predicted ratings.

## 5.2 Experimentation setup

A list of rating predictions to every movie not yet seen by neither user in a group has been generated from individual evaluations  $p(u, i)$  existent in the dataset. The rating predictions were generated by means of a collaborative filtering algorithm based on KNN (*K-Nearest Neighbor*) [7]. Figure 1 shows distribution histogram. Predictions have a mean value of 4.01 and low standard deviation of 0.41.

In order to generate the *optimal coalition structure* ( $CS^*$ ), 10 sets of 12 users of the MovieLens dataset have been randomly generated.  $n = 12$  was the largest number of users with which all method's iterations could be performed in a reasonable time, due to time complexity issue. The value of  $\alpha$  was varied empirically in the range of 0.999 and 1.010 for each iteration.

K-Means [10] has been similarly applied to the same 10 sets of 12 users for comparison purposes. Initial centroids were chosen randomly and  $K$  was varied in the interval of 2 to 6 for each iteration.

## 5.3 Evaluation metrics

A metric for clustering evaluation known as *Davies-Bouldin index* (DB) [8] was used (Equation 9):

$$DB = \frac{1}{n} \sum_{i=1}^n \max_{i \neq j} \left( \frac{\sigma_i + \sigma_j}{d(c_i, c_j)} \right) \quad (9)$$

where  $n$  is the number of clusters,  $c_x$  is the centroid of the cluster  $x$ ,  $\sigma_x$  is the average distance of all the elements in cluster  $x$  to centroid  $c_x$ , and  $d(C_i, j_i)$  is the distance between the centroid  $C_i$  and  $c_j$ .

Good clustering produces clusters with low intra-cluster distance (high internal similarity between cluster elements) and high inter-cluster distances (low similarity between clusters). Better clustering algorithms produces lower *Davies-Bouldin index* values.

## 5.4 Experimentation results

Tables 2 and 3 show the average values of the iterations performed on the 10 sets of users. Tables 4 and 5 show the

Table 2: Results for proposed approach.

$\alpha$	Groups	Satisfaction	DB Index
0.999	6.0	4.5761	1.0755
1.000	5.9	4.5733	1.0904
1.001	5.8	4.5651	1.1077
1.002	5.2	4.5242	1.1977
1.003	4.7	4.4853	1.2315
1.004	3.7	4.3622	1.3108
1.005	3.1	4.2390	1.3971
1.006	2.8	4.1791	1.4306
1.007	2.5	4.1198	1.4601
1.008	2.4	4.0738	1.4587
1.009	2.3	4.0522	1.4479
1.010	2.1	4.0082	1.4370

Table 3: Results for K-Means clustering.

k	Satisfaction	DB Index
2	4.0031	1.5648
3	4.2154	1.4273
4	4.3282	1.3287
5	4.5793	1.1288
6	4.6311	0.9965

correlations between the value of  $K$ , group payoff and DB index values.

We note a strong negative correlation of 0.97 between the  $\alpha$  value and the number of groups provided by the proposed approach, which confirms the rationale of Equation 6 (Table 4).

Table 2 shows that better satisfaction (utility) and DB index values are achieved with lower values of  $\alpha$ , which suggests that smaller groups ensures greater similarity between group members.

For  $K \leq 4$  and  $\alpha \geq 1,004$ , the proposed approach has achieved better results if compared to K-means clustering. This is shown if we compare both satisfaction (utility) and DB index values of last 7 (seven) lines of Table 2 ( $2.1 \leq K \leq 3.7$ , considering the nearest integer) with the first 3 (three) lines of Table 3 ( $2 \leq K \leq 4$ ).

Table 4: Correlation for proposed approach.

		groups	alpha	utility	db index
groups	Corr.	1.00	-0.97	.98	-.99
	Sig.		.00	.00	.00
	N	12	12	12	12
alpha	Corr.	-.97	1.00	-.98	.95
	Sig.	.00		.00	.00
	N	12	12	12	12
utility	Corr.	.98	-.98	1.00	-.96
	Sig.	.00	.00		.00
	N	12	12	12	12
db index	Corr.	-.99	.95	-.96	1.00
	Sig.	.00	.00	.00	
	N	12	12	12	12

**Table 5: Correlation for K-Means clustering.**

		k	utility	db index
k	Corr.	1.00	.97	-1.00
	Sig.		.01	.00
	N	5	5	5
utility	Corr.	.97	1.00	-0.95
	Sig.	.01		.01
	N	5	5	5
db index	Corr.	-1.00	-.95	1.00
	Sig.	.00	.01	
	N	5	5	5

## 6. CONCLUSION

This paper proposes an approach based on the generation of *coalitional structures* in a cooperative game to maximize the total *social welfare* of the groups of users formed in the context of group recommender systems. Coalitions formation is performed by means of a *payoff* function which considers the similarity between members of the coalition, concerning items ratings previously provided in the recommendation system, and a weighting factor for the coalition size.

Experiments with a MovieLens dataset were used to compare this approach with the state-of-the-art K-Means clustering. The results show better performance of the proposed approach in the formation of larger coalitions, considering two different metrics: the average group satisfaction and *Davies-Bouldin index*, which is commonly used to evaluate mainstream clustering algorithms. Furthermore, the approach does not depend on previously established *K* value to provide clustering, an important known limitation of K-Means.

Results show the feasibility of the proposal to provide group formation for further recommendation tasks in contexts where there are no reason to predefine groups.

Future work intend to extend experimentation with real users of the MyPopCorn<sup>2</sup> recommendation system, an application that runs on Facebook. Friendship levels between group members might be used to compute the payoff of the coalition structure.

Another future work is related to the comparison of the approach proposed in this paper with other clustering methods such as the Hierarchical Clustering [6].

Finally, a further work intends to deal with the fairness of the recommendation provided to the formed group. The research hypothesis is that it could be done by finding the set of items to be recommended that better approximates the *the Shapley value* of each group member.

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<sup>2</sup><http://mypopcorn.info>

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